$$\int_{-1}^{1} \frac{x^9}{\sqrt{1-x^2}}\,dx$$

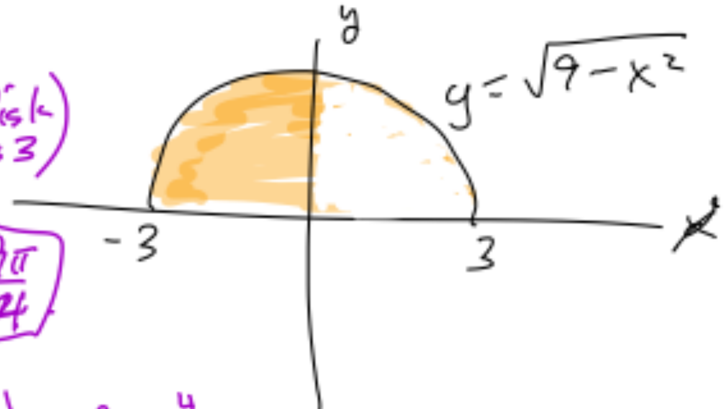
Similar questions

① $$\int_{-3}^{0} \sqrt{9-t^2}\,dt$$

$y^2 = 9 - x^2 \Leftrightarrow x^2+y^2=9$

$= \frac{1}{4}$(area of disk of radius 3)

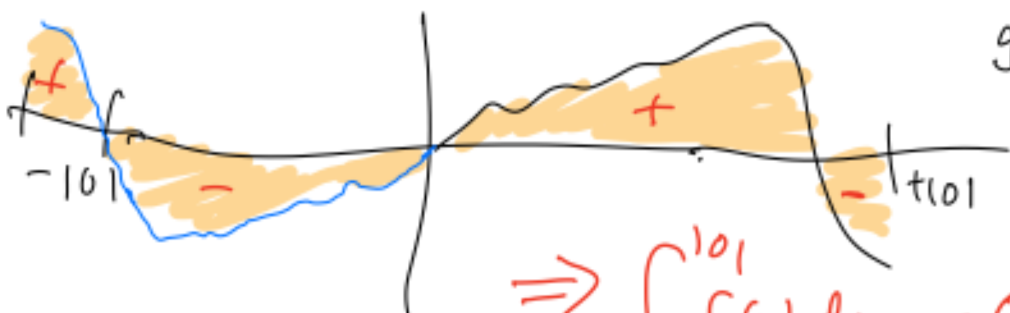
$= \frac{1}{4}\pi(3)^2 = \boxed{\frac{9\pi}{4}}$



② $$\int_{-101}^{101} \underbrace{e^{3x^2-x^4}\cos(2x)\sin(x^3)}_{f(x)}\,dx = ?$$

Notice $f(-x) = -f(x)$ "odd fcn"

$g(-x) = g(x)$ "even fcn"



$$\Rightarrow \int_{-101}^{101} f(x)\,dx = 0.$$

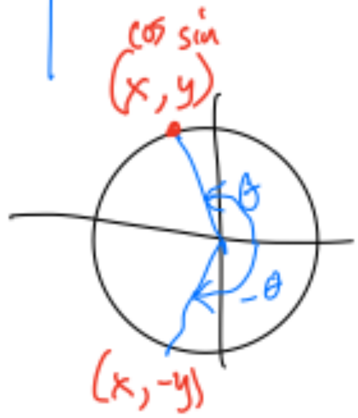
Examples of odd & even fcns:

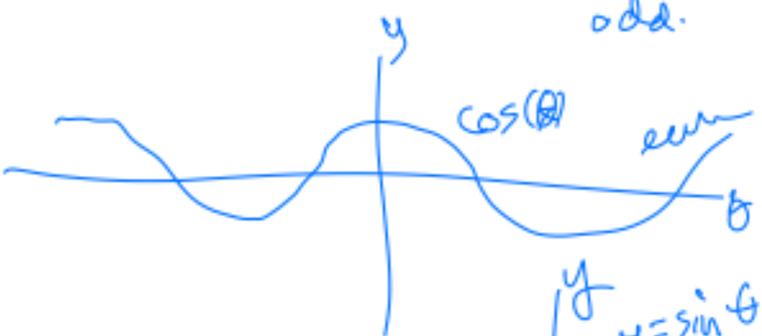
① x^5 odd since $(-x)^5 = -x^5$

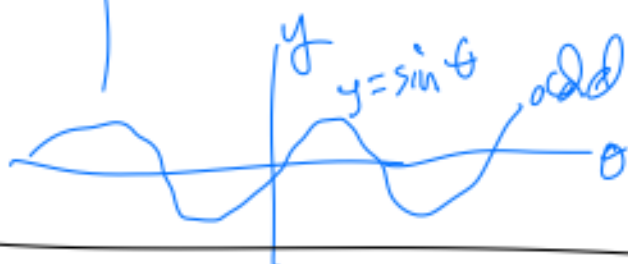
x^8 even since $(-x)^8 = x^8$

② $\cos(-\theta) = \cos(\theta)$
even

$\sin(-\theta) = -\sin\theta$
odd.







ⓛ $\int \frac{1}{\sqrt{\sqrt{x}+1}}\,dx$

Similar: $\int (2-\sqrt{x})^{-3/2}\,dx = \int \frac{1}{(2-\sqrt{x})^{3/2}}\,dx$

Let $u = 2 - \sqrt{x}$

$\cancel{du = -\frac{1}{2}x^{-1/2}dx}$ Not there in integral

$$\sqrt{x} = 2 - u$$

$$x = (2-u)^2$$

$$dx = -2(2-u)\,du$$

$$= \int \frac{1}{u^{3/2}}\left(-2(2-u)\,du\right)$$

$$\left[(2-u)^2\right]' = 2(2-u)(2-u)'$$
$$= 2(2-u)(-1)$$
$$= -2(2-u)$$

$$= \int \frac{1}{u^{3/2}}(-4+2u)\,du$$

$$= \int -4u^{-3/2} + 2u^{-1/2}\,du$$

$$\int u^{-3/2}\,du$$
$$= \frac{u^{-3/2+1}}{-3/2+1} + C$$
$$= \frac{u^{-1/2}}{-1/2} + C$$
$$= (-2)u^{-1/2} + C$$

$$= (-4)(-2)u^{-1/2} + 2(2)u^{1/2} + C$$

$$= 8u^{-1/2} + 4u^{1/2} + C$$

$$= \boxed{8(2-\sqrt{x})^{-1/2} + 4(2-\sqrt{x})^{1/2} + C}$$

Back to our practice integrals:

④ $\displaystyle\int_0^1 \frac{1}{2x^2+2x+1}\,dx$

$$\text{denom} = 2x^2+2x+\tfrac{1}{2}+\tfrac{1}{2}$$

$$= \tfrac{1}{2}\left(4x^2+4x+1+1\right)$$

$$= \tfrac{1}{2}\left((2x+1)^2+1\right)$$

Completing the Square

We can let $u=(2x+1)$ $du=2dx$

$\frac{1}{2}du = dx$

$$= \int_{\substack{x=0\\u=1}}^{\substack{u=3\\1}} \left(\frac{1}{\frac{1}{2}(u^2+1)}\right)\frac{1}{2}\,du = \int_{u=1}^{3} \frac{1}{u^2+1}\,du$$

$$= \arctan(u)\Big|_1^3 = \arctan(3) - \underbrace{\arctan(1)}_{\frac{\pi}{4}}$$

$$= \boxed{\arctan(3) - \pi/4}$$

Another way to complete the square $\quad Ax^2+Bx+C = A(x+\text{[illegible]})^2+\text{[illegible]}$

$$2x^2+2x+1 = 2(x^2+x)+1$$

$$= 2(x^2+x+\_\_)+1-2\_\_$$

[Idea $\quad x^2+bx+\overset{(\frac{b}{2})^2}{\frac{b^2}{4}} = (x+\frac{b}{2})^2$]

$$= 2\left(x^2 + x + \frac{1}{4}\right) + \underbrace{1 - 2\left(\frac{1}{4}\right)}_{\frac{1}{2}}$$

$$= 2\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{2}$$

$$= 2\left(x + \frac{1}{2}\right)^2 + \frac{1}{2}$$

$$\frac{1}{2\left(x+\frac{1}{2}\right)^2 + \frac{1}{2}} = \frac{1}{\frac{1}{2}\left(\underbrace{4\left(x+\frac{1}{2}\right)^2}_{(2x+1)^2} + 1\right)}$$

$u = 2x+1$

Technique of Substitution:

Reverse of the chain rule.

$$\left[f(u(x))\right]' = f'(u(x)) \cdot u'(x)$$

$$\left[f(u(x))\right]' dx = f'(u) \underbrace{du}_{u'(x)\,dx.}$$

Another technique of finding antiderivatives is <u>integration by parts</u> (reverse of the product rule).

<u>Product Rule</u>: If u & v are functions that are differentiable, then $(uv)' = u'v + uv'$

$$(u(x)v(x))' = u'(x)\,v(x) + u(x)\,v'(x)$$

$$\int (uv)'\,dx = \int u'v\,dx + \int u\,\underbrace{v'\,dx}_{\underbrace{v'(x)\,dx}_{dv}}$$

.... to be continued